APPENDIX A
EQUIVALENCE OF $cost_{SumMax2}$ AND $cost_{Sum}$

We have the following lemma to show the functionality of $cost_{SumMax2}$ and $cost_{Sum}$ are equivalent.

Lemma 4. Let S be an object set. $cost_{SumMax2}(S) = cost_{Sum}(S)$. □

Proof. Let $(o_1, o_2) = \arg\max_{o_1,o_2 \in S} d(o_1, o_2)$. We have $cost_{SumMax2}(S) = \max\{\sum_{o\in S} d(o,q), d(o_1,o_2)\}$. Note that $d(o_1,q) + d(o_2,q) \geq d(o_1,o_2)$ by triangle inequality and $\sum_{o\in S} d(o,q) \geq d(o_1,q)+d(o_2,q)$. Thus, $cost_{SumMax2}(S) = \sum_{o\in S} d(o,q) = cost_{Sum}(S)$. □

This lemma suggests that it is sufficient to consider one of these two cost functions. In this paper, we focus the discussion on $cost_{Sum}$.

APPENDIX B
PROOF OF THEOREM 1

We first give the decision problem of CoSKQ. Given a set O of spatial objects each $o \in O$ associated with a location $o.\lambda$ and a set of keywords $o.\psi$, a query q consisting of a query location $q.\lambda$ and a set of query keywords $q.\psi$, and a real number C, the problem is to determine whether there exists a set S of objects in O such that S covers the query keywords and $cost_{unified}(S)$ is at most C.

We then prove by transforming the *3-satisfiability* (3-SAT) problem which is known to be NP-Complete to the CoSKQ problem and showing the equivalence between two problems. The description of the 3-SAT problem is given as follows. Let U be a set of literals $\{e_1, \overline{e_1}, ..., e_n, \overline{e_n}\}$ where $\overline{e_i}$ is the negation of e_i. Given an expression $E = C_1 \wedge C_2 \wedge ... \wedge C_m$ where $C_j = x_j \vee y_j \vee z_j$ and $x_j, y_j, z_j \in U$ for $1 \leq j \leq m$, the problem is to determine whether there exists a truth assignment for e_i for $i \leq i \leq n$ such that E is true.

Based on the value of parameter ϕ_1, we use different transformations.

Case 1. $\phi_1 = 1$**.** We construct a set O of $2n$ objects as follows. For each literal e_i in U, we create an object o_i in O, and for each literal $\overline{e_i}$ in U, we create an object o'_i in O. In total, there are $2n$ objects in O. We set the locations of the objects in O such that they are all located at the same place i.e., for any $o \in O$, $o.\lambda$ is identical. Besides, for each object o_i ($1 \leq i \leq n$), we set $o_i.\psi$ such that $o_i.\psi$ includes a keyword k_i corresponding to e_i and a keyword k'_j corresponding to C_j if C_j involves e_i for $1 \leq j \leq m$. Similarly, for each object o'_i ($1 \leq i \leq n$), we set $o'_i.\psi$ such that $o'_i.\psi$ includes the keyword k'_i and all k'_j's with C_j involving $\overline{e_i}$ for $1 \leq j \leq m$. We construct a query q by setting $q.\lambda$ to be a location such that $d(o,q) = 1$ for any object $o \in O$ and $q.\psi$ to be a set of $m+n$ keywords, $\{k_1, k_2, ..., k_n, k'_1, k'_2, ..., k'_m\}$. We set C to be n. The above transformation process could be done in polynomial time.

Case 2. $\phi_1 \in \{\infty, -\infty\}$**.** We construct a set O of $2n$ objects as follows. For each literal e_i in U, we create an object o_i in O, and for each literal $\overline{e_i}$ in U, we create an object o'_i in O. In total, there are $2n$ objects in O. For the locations of the objects, consider a circle Cir with its center at $q.\lambda$ (which is selected arbitrarily) and its radius equal to 1. We set the locations of the objects in O such that they are all located on the boundary of Cir such that $d(o_i, o'_i) = 2$. Besides, for each object o_i ($1 \leq i \leq n$), we set $o_i.\psi$ such that $o_i.\psi$ includes a keyword k_i corresponding to e_i and a keyword k'_j corresponding to C_j if C_j involves e_i for $1 \leq j \leq m$. Similarly, for each object o'_i ($1 \leq i \leq n$), we set $o'_i.\psi$ such that $o'_i.\psi$ includes the keyword k'_i and all k'_j's with C_j involving $\overline{e_i}$ for $1 \leq j \leq m$. We construct a query q by setting $q.\lambda$ arbitrarily and $q.\psi$ to be a set of $m+n$ keywords, $\{k_1, k_2, ..., k_n, k'_1, k'_2, ..., k'_m\}$. The above transformation process could be done in polynomial time. We consider the following sub-cases for setting C.

Case 2(a). $\phi_2 = 1$**.** We set $C = 3 - \epsilon$ where ϵ is close to zero.

Case 2(b). $\phi_2 = \infty$**.** We set $C = 2 - \epsilon$ where ϵ is close to zero.

We show the equivalence between two problem instances as follows. Suppose that the answer of the 3-SAT problem is "yes", i.e., there exists a truth assignment for the literals in U such that E is correct. We denote the truth assignment by a set T of literals which are true under the assignment. Note that T has exactly n literals and e_i and $\overline{e_i}$ do not appear in T simultaneously for any $1 \leq i \leq n$. Then, it could be verified that the set of objects each corresponding to a literal in T covers $q.\psi$ and the cost of the set at most C, and thus the answer of the CoSKQ problem is also "yes". Suppose that the answer of the CoSKQ problem is "yes". Let S be the set of objects in O that covers $q.\psi$ and has the cost at most C. We know that object o_i and o'_i are not included in S simultaneously. It could be verified that with the truth assignment represented by the set of literals corresponding to the objects in S, E is correct, and thus the answer of the 3-SAT problem is also "yes". □

APPENDIX C
PRUNING BASED ON DOMINANCE

To improve the efficiency of the algorithm, we propose a pruning strategy to prune the search space when $\alpha = 1$ and $\phi_1 = 1$. Before we give the strategy, we first introduce the concept of **dominance**. Given a query q, two objects o_1 and o_2, we say o_1 dominate o_2 if the following two conditions are satisfied. (1) $d(o_1,q) < d(o_2,q)$, and (2) all keywords in $q.\psi$ that are covered by o_2 can be covered by o_1, (i.e. $q.\psi \cap o_1.\psi \supseteq q.\psi \cap o_2.\psi$). A dominant object is defined to be an object that is not dominated by any other objects. Then we have the following lemma to prune the objects that are not dominant objects.

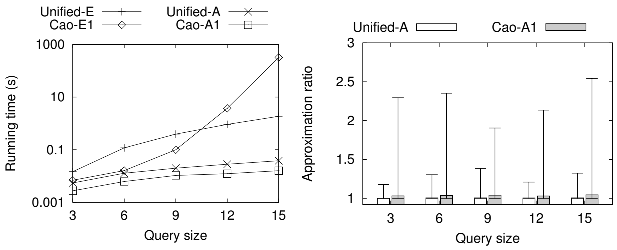
Lemma 5. When $\alpha = 1$ and $\phi_1 = 1$, all objects in the optimal solution S are dominant objects. □

Proof: We prove this by contradiction. Let an object $o \in S$ that is not a dominant object. Then, there must exist an object o' that dominate o. Note that o' also covers the query keywords covered by o and is closer to q. We can construct a better solution $S' = S \setminus \{o\} \cup \{o'\}$, which contradicts the fact that S is the optimal solution. □

Based on this lemma, it is sufficient for the algorithm to consider the dominant objects only when enumerating the object sets. Specifically, whenever the algorithm performs a range query, it discards the objects that are being dominated and proceeds with the dominant objects.

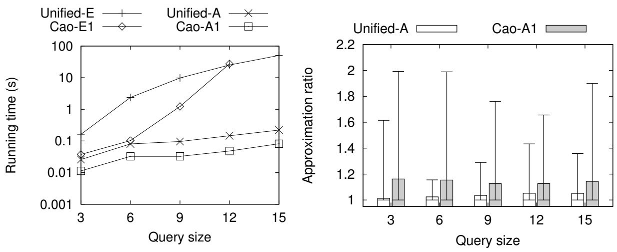
APPENDIX D
BETTER IMPLEMENTATION BASED ON INFORMATION RE-USE

To implement the *Unified-A* algorithm efficiently, we have the following computation strategies. First, when the algorithm finding



(a) Running time (b) Approximation ratio

Fig. 18: Effect of $|q.\psi|$ on $cost_{MinMax}$ (GN)



(a) Running time (b) Approximation ratio

Fig. 19: Effect of $|q.\psi|$ on $cost_{MinMax}$ (Web)

the set of all relevant objects in R_o (line 5 in Algorithm 4), instead of issuing a range query in each iteration, it re-uses the information from the previous iteration by maintaining the region R_o dynamically. Specifically, consider one iteration. The algorithm finds a feasible set that has an object o as a key query-object distance contributor in the region R_o. After it finishes the current iteration, it adds o into R_o (when $\phi_1 \in \{1, \infty\}$), or removes o from R_o (when $\phi_1 = -\infty$).

Second, when the algorithm performs the iterative process (lines 8-14 in Algorithm 4), instead of searching for the object with minimum ratio (distance) from $\mathcal{O}'$ in each iteration, it maintains a heap structure for storing the objects. Specifically, when $\phi_1 = 1$, the key of the objects in the heap are the ratios, and the heap is updated after each object is picked. When $\phi_1 \in \{\infty, -\infty\}$, the key of the objects in the heap are the distances, and in each iteration the algorithm picks the relevant object with the smallest distance.

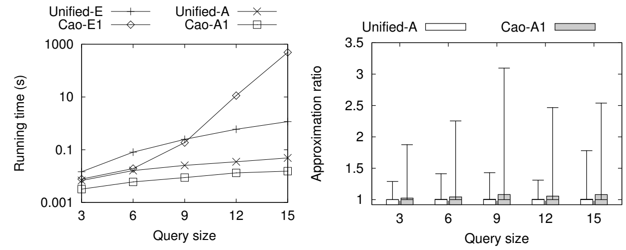
Appendix E Experimental Results on the datasets GN and Web

In the following, we present the experimental results on the datasets GN and Web of varying $|q.\psi|$. Following the existing studies [3], [17], we vary the number of query keywords (i.e., $|q.\psi|$) from $\{3, 6, 9, 12, 15\}$.

(1) $cost_{MinMax}$**.** The results for $cost_{MinMax}$ on the datasets GN and Web are shown in Figure 18 and Figure 19, respectively, which are similar to that on the dataset Hotel. The result of running time of *Cao-E1* for $|q.\psi| = 15$ is not shown in Figure 19 simply because it ran for more than 10 hours (this applies for all the following results).

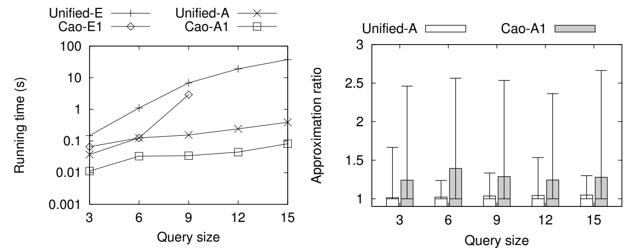
(2) $cost_{MinMax2}$**.** The results for $cost_{MinMax2}$ on the datasets GN and Web are shown in Figure 20 and Figure 21, respectively, which are similar to those for $cost_{MinMax}$.

(3) $cost_{Sum}$**.** The results for $cost_{Sum}$ on the datasets GN and Web are shown in Figure 22 and Figure 23, respectively. According to the results, *Unified-E* runs slower than *Cao-E2* but still within a reasonable time (e.g. within 10 seconds on the largest dataset



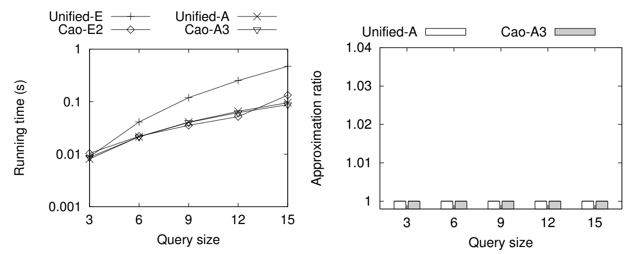
(a) Running time (b) Approximation ratio

Fig. 20: Effect of $|q.\psi|$ on $cost_{MinMax2}$ (GN)



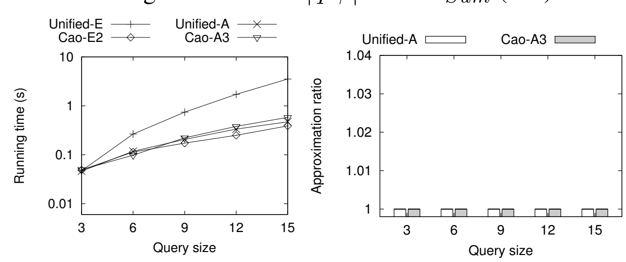
(a) Running time (b) Approximation ratio

Fig. 21: Effect of $|q.\psi|$ on $cost_{MinMax2}$ (Web)



(a) Running time (b) Approximation ratio

Fig. 22: Effect of $|q.\psi|$ on $cost_{Sum}$ (GN)



(a) Running time (b) Approximation ratio

Fig. 23: Effect of $|q.\psi|$ on $cost_{Sum}$ (Web)

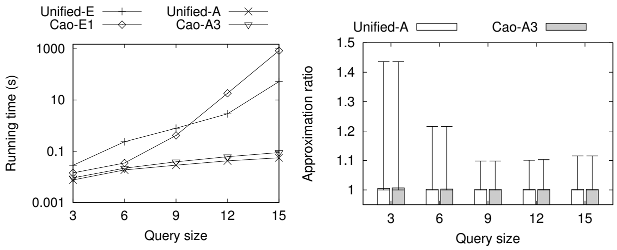
Web). Besides, *Unified-A* has a very similar running time as *Cao-A3*, while *Unified-A* can always obtain an approximation ratios of 1.

(4) $cost_{SumMax}$**.** The results for $cost_{SumMax}$ on the datasets GN and Web are shown in Figure 24 and Figure 25, respectively, which are similar to that on the dataset Hotel.

(5) $cost_{MaxMax}$**.** The results for $cost_{MaxMax}$ on the datasets GN and Web are shown in Figure 26 and Figure 27, respectively, which are similar to that on the dataset Hotel.

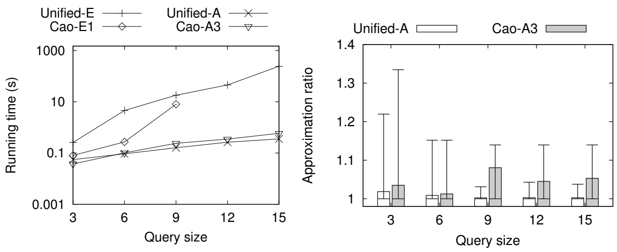
(6) $cost_{MaxMax2}$**.** The results for $cost_{MaxMax2}$ on the datasets GN and Web are shown in Figure 28 and Figure 29, respectively, which are similar to that on the dataset Hotel.

(7) $cost_{Max}$**.** The results for $cost_{Max}$ on the datasets GN and Web are shown in Figure 30, which is similar to that on the dataset Hotel. According to the results, both *Unified-E* and *Unified-A* run very fast, e.g. they ran less than 6 ms for all settings of $|q.\psi|$.



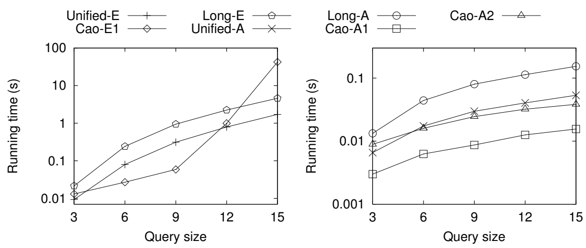
(a) Running time (b) Approximation ratio

Fig. 24: Effect of $|q.\psi|$ on $cost_{SumMax}$ (GN)

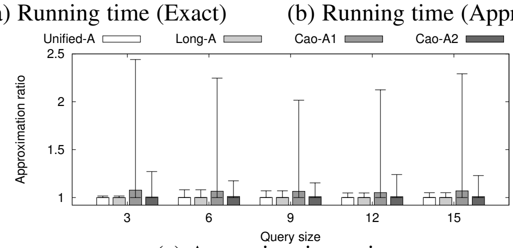


(a) Running time (b) Approximation ratio

Fig. 25: Effect of $|q.\psi|$ on $cost_{SumMax}$ (Web)

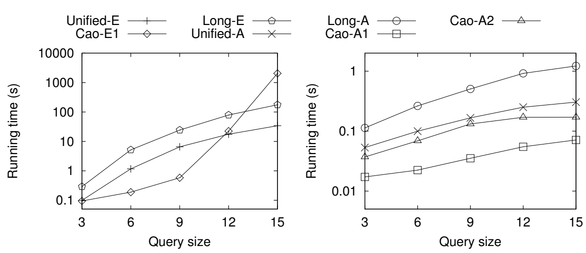


(a) Running time (Exact) (b) Running time (Appro.)

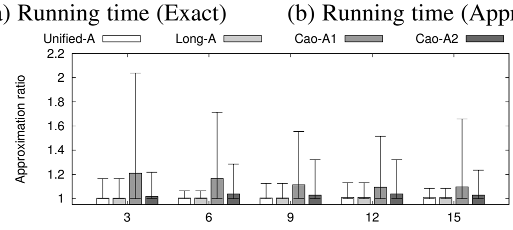


(c) Approximation ratio

Fig. 26: Effect of $|q.\psi|$ on $cost_{MaxMax}$ (GN)



(a) Running time (Exact) (b) Running time (Appro.)



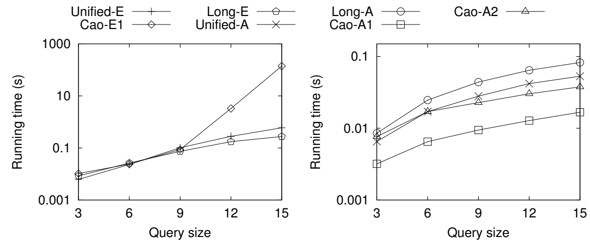
(c) Approximation ratio

Fig. 27: Effect of $|q.\psi|$ on $cost_{MaxMax}$ (Web)

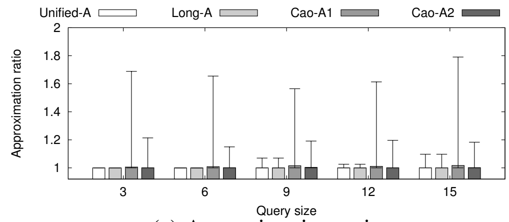
Appendix F
Scalability Test

(2) $cost_{MinMax2}$**.** The results for $cost_{MinMax2}$ are shown in Figure 31. According to Figure 31(a), *Unified-E* is faster and more scalable than *Cao-E1*, e.g., on a dataset with 6M objects, *Unified-E* ran for a couple of seconds while *Cao-E1* ran for more than 10 hours. Besides, similar to the case of $cost_{MinMax}$, *Unified-A* runs slightly slower than *Cao-A1*, but gives much better approximation ratio, e.g. the median of approximation ratios of *Unified-A* are 1 on all settings while that of *Cao-A1* are larger than 1.

(3) $cost_{Sum}$**.** The results for $cost_{Sum}$ are shown in Figure 32. According to Figure 32(a), *Unified-E* is very scalable when the number of objects is large, e.g., it ran slightly longer than 1 second on a dataset with 10M objects. Besides, we noticed that *Cao-E2* has a very good performance and it even runs as fast as the approximation algorithms. The reason could be as follows. With the number of objects grows, the number of relevant objects

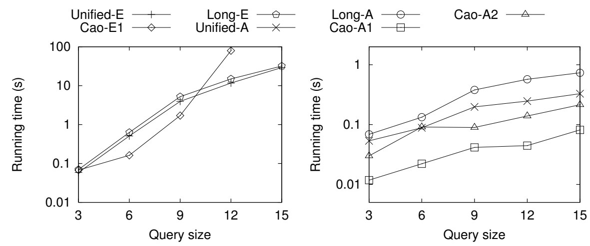


(a) Running time (Exact) (b) Running time (Appro.)

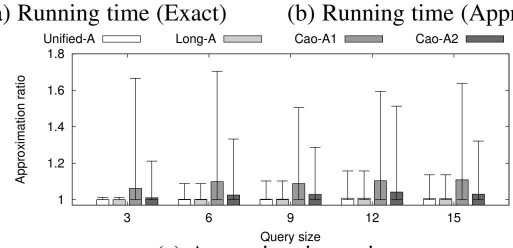


(c) Approximation ratio

Fig. 28: Effect of $|q.\psi|$ on $cost_{MaxMax2}$ (GN)

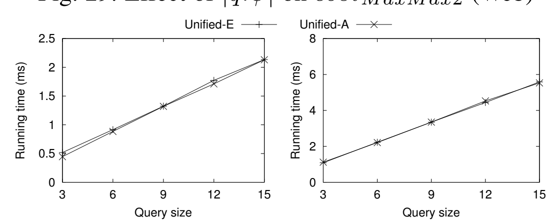


(a) Running time (Exact) (b) Running time (Appro.)



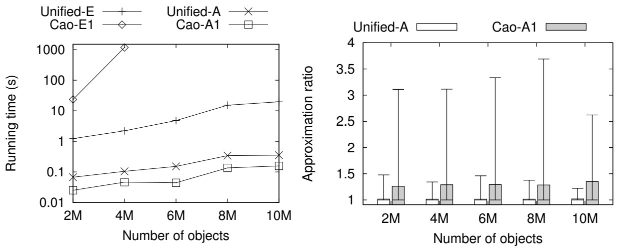
(c) Approximation ratio

Fig. 29: Effect of $|q.\psi|$ on $cost_{MaxMax2}$ (Web)



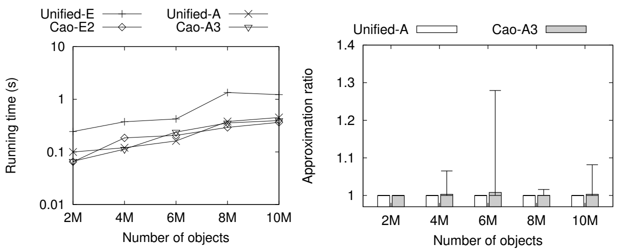
(a) GN (b) Web

Fig. 30: Effect of $|q.\psi|$ on $cost_{Max}$



(a) Running time (b) Approximation ratio

Fig. 31: Scalability test on $cost_{MinMax2}$

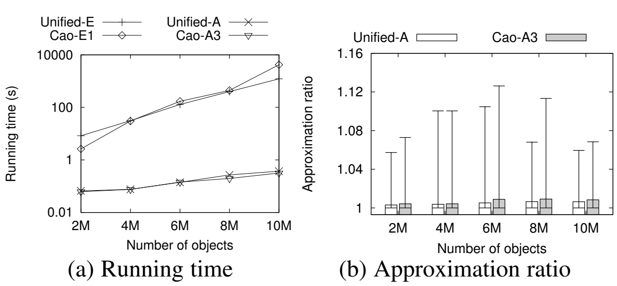


(a) Running time (b) Approximation ratio

Fig. 32: Scalability test on $cost_{Sum}$

becomes large. Both approximate algorithms have to re-compute the ratio for the remaining nodes in the heap and re-organize the heap after picking each object, whose cost becomes expensive when the number of relevant objects is large. In contrast, *Cao-E2* maintains a heap structure though, it does not have to re-examine the nodes after processing a node. *Unified-A* has similar running times as *Cao-A3* but gives better approximation ratios than *Cao-A3* (Figure 32(b)). Specifically, *Unified-A* can achieve near-to-optimal approximation ratios on all setting while *Cao-A3* has its largest approximation ratios up to 1.279.

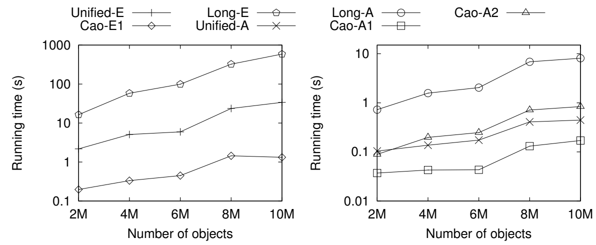
(4) $cost_{SumMax}$**.** Same as the experiments of varying $|o.\psi|$ for $cost_{SumMax}$, we used the setting of $|q.\psi| = 8$ for the scalability test experiments for $cost_{SumMax}$ particularly. The results for $cost_{SumMax}$ are shown in Figure 33. According to Figure 33(a), *Unified-E* and *Cao-E1* have similar running times and *Unified-A* and *Cao-A3* also have similar running times, but *Unified-A* gives a better approximation ratio than *Cao-A3* (Figure 33(b)).



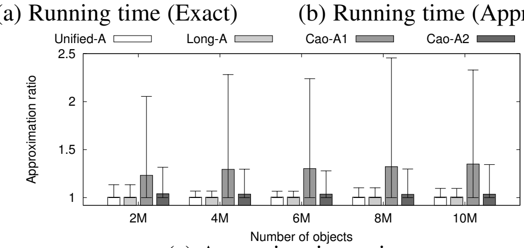
(a) Running time (b) Approximation ratio

Fig. 33: Scalability test on $cost_{SumMax}$

(5) $cost_{MaxMax}$**.** The results for $cost_{MaxMax}$ are shown in Figure 34. According to Figure 34(a), *Unified-E* runs faster than *Long-E* but slower than *Cao-E1*. According to Figure 34(b) and (c), *Unified-A* runs faster than *Long-A* and *Cao-A2* and slower than *Cao-A1*, and *Unified-A* is one of the two algorithms (the other is *Long-A* which runs slower than *Unified-A* by about one order of magnitude) which give the best approximation ratios. Specifically, the largest approximation ratios of *Unified-A* is only 1.134, which is small, while that of *Cao-A1* and *Cao-A2* are 2.456 and 1.345, respectively.



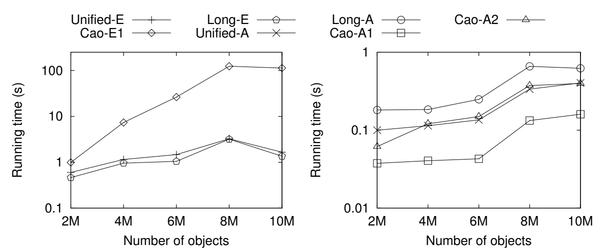
(a) Running time (Exact) (b) Running time (Appro.)



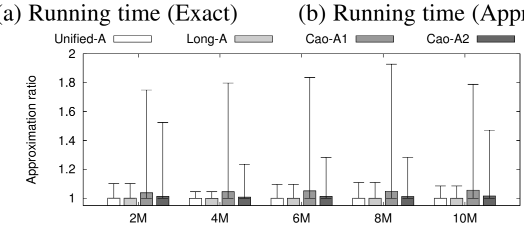
(c) Approximation ratio

Fig. 34: Scalability test on $cost_{MaxMax}$

(6) $cost_{MaxMax2}$**.** The results for $cost_{MaxMax2}$ are shown in Figure 35. According to Figure 35(a), *Unified-E* runs similarly fast as *Long-E*, and both of them run faster than *Cao-E1*. According to Figure 35(b) and (c), *Unified-A* has similar running times with *Cao-A2*, both of them run faster than *Long-A* and slower than *Cao-A1*, and *Unified-A* is one of the two algorithms (the other is *Long-A* which runs slower than *Unified-A*) which give the best approximation ratios. Specifically, the largest approximation ratios of *Unified-A* is only 1.109, which is small, while that of *Cao-A1* and *Cao-A2* are 1.928 and 1.524, respectively.



(a) Running time (Exact) (b) Running time (Appro.)



(c) Approximation ratio

Fig. 35: Scalability test on $cost_{MaxMax2}$

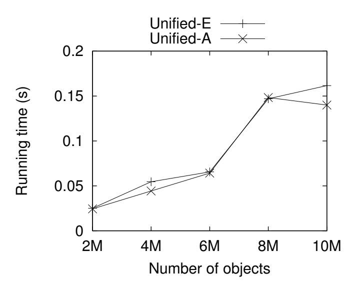


Fig. 36: Scalability test on $cost_{Max}$

(7) $cost_{Max}$**.** The results for $cost_{Max}$ are shown in Figure 36. According to the results, both *Unified-E* and *Unified-A* runs very fast, e.g. they ran within 1 second on a dataset with 10M objects.